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Art. XV.—*Variation of Wind with Height at Melbourne when Geostrophic Winds are Northerly.*

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The geostrophic wind is an ideal wind blowing along the isobars with, in the Southern hemisphere, low pressure on its right. Its velocity G is defined by the equation—

$$2\omega \sin\phi . G = \frac{1}{\rho} \frac{dp}{dn} \qquad . \quad . \quad . \quad . \quad . \qquad A$$

where ϕ is the latitude,
ρ is the density of air,
ω is the angular velocity of rotation of the earth,
$\frac{dp}{dn}$ is the pressure change per unit distance normal to the isobars.

It has been shown that in other parts of the world the surface geostrophic wind is usually a good approximation to the actual wind at a height varying from about 1500 ft. to 3000 ft. The equation shows that the surface geostrophic wind can be computed from the surface isobars. Using data for 1922, G was calculated for Melbourne when the geostrophic wind direction was between N.22½°E. and N.22½°W., only those days being used for which a good determination of the pressure gradient could be made, and for which pilot balloon observations of the wind up to at least a kilometre were available. The accuracy of the wind determinations in the first half kilometre is high since distances are determined directly by rangefinder, thus avoiding the assumption of a uniform rate of ascent of the balloon. Both the 9 hour and 15 hour daily pressure charts were utilised, but the 9 hour determinations were by far the more numerous. The interval of time separating pressure and wind observations amounted to only half an hour.

The investigations of this paper are confined to geostrophic winds of less than 13 metres per second. Since two-thirds of the values lie between 7·4 and 10·5 metres per second the conditions are fairly homogeneous. For higher velocities the conditions are more complex and their investigation has not been completed. Means of the actual and geostrophic winds are exhibited in Table I. and Figure 1. This paper is concerned with the explanation of the average actual wind distribution from near the surface up to about 1000 metres in terms of the eddy diffusivity and geostrophic wind.

TABLE I.

Height above Theodolite	Variation of Wind with Height			
	Observed		Computed	
	V	θ	V	θ
m	m/s	°	m/s	°
50	5·5	18·6	5·6	18·5
150	7·3	16·2	7·2	16·6
300	8·8	11·0	8·6	11·3
450	9·3	7·0	9·2	6·5
750	9·7	2·2	9·3	0.9
1000	9·0	− 7	9·1	− 0·4
1300	10·0	− 17		
1500	9·4	− 21		
2000	10·4	− 26		

Mean Geostrophic Wind 8·9 m/s. θ is the deviation of the wind from the surface isobars.

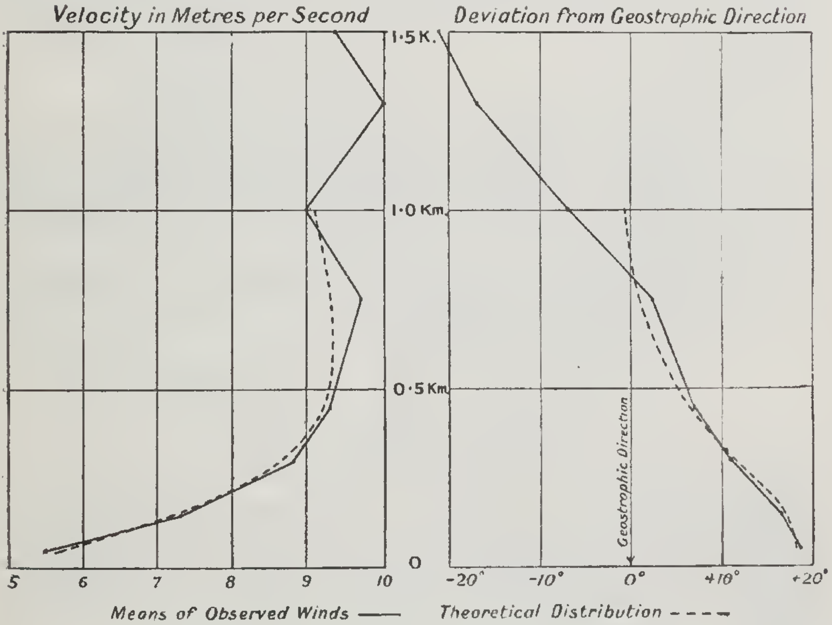

FIG. 1.—Variation of Wind with Height.

Air in uniform horizontal motion is under balanced forces due to gravity, pressure, friction and the earth's rotation. The equations of motion (1) referred to rectangular axes rotating with the earth are:—

$$-K\rho\frac{d^2u}{dz^2}=2\omega v\rho\sin\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad B$$

$$-K\rho\frac{d^2v}{dz^2}=-2\omega u\rho\sin\phi+2\omega G\rho\sin\phi \quad . \quad . \quad . \quad C$$

The motion at any height z is indicated in Figure 2, where OX is parallel to the surface geostrophic wind and OY is in the direction of decreasing pressure. The angle GOE is the angle between the actual wind and the geostrophic wind. The component wind velocities u and v are along OX, OY respectively.

Brunt simplified the treatment of the equations by using a vectorial notation, representing the horizontal velocity by W where $W = u + iv$.

Then regarding K and G as constants the solution, after neglecting that part which leads to an indefinite increase of velocity with height, is given by him (2) as

$$W - G = Ce^{-(1+i)Bz + i\gamma} \quad . \quad . \quad . \quad . \quad . \quad D$$

where C and γ are real constants to be determined from boundary conditions and $B^2 = \omega \sin\phi / K$.

Brunt, following Taylor and others, assumes that at the ground the up-gradient of velocity is in the direction of motion. In theory this condition is applied at the surface, but in practice $z=0$ is taken to be some indefinite height "near the surface." This is equivalent to applying at a height near the surface the determining condition that there the up-gradient of velocity is in the direction of motion.

Three objections may be raised to these conditions:—

1. They are based upon an assumption which certainly does not always hold. Richardson (3) for instance gives a number of examples showing that the stress on the ground has a direction differing from the wind near the surface.
2. The height at which they are applied is too indefinite. This objection is important since the velocity changes rapidly near the surface.
3. It is now known that near the surface the eddy diffusivity decreases rapidly in disagreement with the assumption used in solving the equations.

An improvement would therefore be made if a condition for determining the constants could be found which was applicable at a definite height and required no additional assumption regarding frictional forces. It would be a further gain if this height were such that the condition as to constancy of eddy diffusivity was approximately fulfilled. Equation D leads to the following result:—

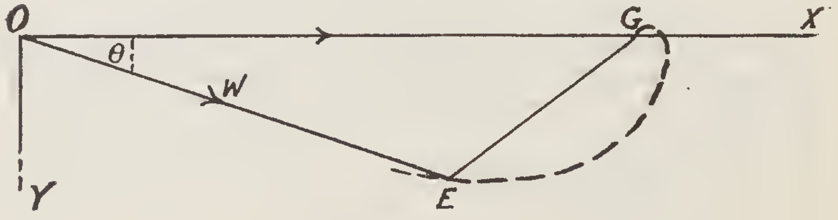

Fig. 2.

Referring to the explanatory diagram, Fig. 2, OG represents the geostrophic velocity along OX, and OE the vector velocity W. It has been pointed out that as W varies with height the solution indicates that the end of the vector GE traces out an equiangular spiral. Now if at any height z_1, the vector representing the wind is a tangent to that spiral, then at that height the direction of shear is in the direction of motion. The usual conditions will then be applicable at z_1, no additional assumption being needed. Since Brunt used the same conditions for different reasons as applying at the surface, his results can be used by substituting $z-z_1$ for his z. Hence the final solution is (2)—

$$W-G=\sqrt{2}G\sin\theta_1 e^{-B(z-z_1)+i[\theta_1+\frac{3}{4}\pi-B(z-z_1)]} \quad . \quad . \quad \text{E}$$

and at z_1, the well-known relation holds

$$\sqrt{u^2+v^2}=V=G(\cos\theta_1-\sin\theta_1) \quad . \quad . \quad . \quad . \quad \text{F}$$

θ_1 is the deviation of the wind from the geostrophic direction at the height z_1.

The value of K may be obtained from the wind data by the use of the formula

$$B^2=\frac{\left(\frac{du}{dz}\right)^2+\left(\frac{dv}{dz}\right)^2}{2\{(G-V)^2+2G(V-u)\}} \quad . \quad . \quad . \quad . \quad \text{G}$$

This follows immediately from equation D, and the constancy of G and K is therefore assumed. For the intervals 50-150 metres, and 150-300 metres, the magnitudes of K were found to be $3{\cdot}1.10^4$ and $3{\cdot}7.10^4$ respectively. Beyond that height reliance cannot be placed on the results as the quantities involved become too small. With the same assumption that G and K are constant, the latter can also be obtained from the relations given by Taylor (4)—

$$BH_d=\tfrac{3}{4}\pi+\theta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{H}$$

$$e^{-BH_v}=\frac{(1+\tan\theta_1)\cos BH_v-(1-\tan\theta_1)\sin BH_v}{\tan\theta_1} \quad \ldots \quad \text{I}$$

where

H_d=height at which the geostrophic direction is reached.
H_v=height at which the geostrophic velocity is reached.

For the winds dealt with here H_d=825m, H_v=340m, $\theta_1=18{\cdot}6°$.

The resulting values of K are $4{\cdot}8.10^4$ and $4{\cdot}2.10^4$ respectively. The eddy diffusivity thus appears to be approximately constant over the range of height considered and a good average value is $4{\cdot}0.10^4$.

Theoretically, since θ_1 is the maximum value of θ, the deviation from the geostrophic direction, it should be possible to determine θ_1 and consequently z_1, directly from the observations. Since θ_1 varies very slowly with height near z_1, and owing to various sources of irregularity, the results so obtained would be rather indefinite. The approximate value of θ_1 from the observations is 18·6°. Now it can be shown that

$$\text{Angle OGE (Fig. 2)} = B(z - z_1) - \theta_1 + \pi/4$$

Hence knowing θ_1, z_1 can be computed from the observations at various heights. Using the above approximate value of θ_1, we obtain the following results:—

$z =$	50	150	300	450	750 metres.
$z_1 =$	54	40	15	34	80 metres.

These give a mean value of 45 m. for z_1.

The value 50 m. will be adopted for z_1. It is near that determined above from the 50 m. observations for which the probable error is least, and also near the mean value. Furthermore it is the height at which θ_1 is a maximum and the equation F is fulfilled by the observations at that height. At 50 m. above the ground the effect of the decreasing eddy diffusivity will be less than at the lower heights usually taken as " near the surface."

We now have for the theoretical distribution the constants—

$\theta_1 = 18{\cdot}6°$, $z_1 = 50$ m., $G = 8{\cdot}9$ m/s and $K = 4{\cdot}0.10^4$.

The values of V and θ derived from equation E are shown for stated heights in Table I. and graphed in Fig. 1 along with the actual winds, with which they are in remarkable agreement. Most of the calculations in this paper were made with a M.O. Pilot Balloon Slide Rule.

In conclusion I wish to thank Mr. H. A. Hunt, Commonwealth Meteorologist, for facilities for writing this paper.

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